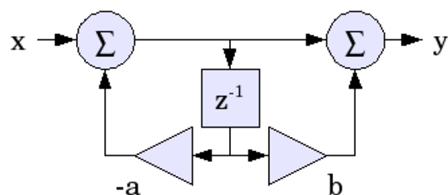


Test 3: Discrete Time LTI Systems and Z-Transforms

- A system is governed by the difference equation $y[n] + y[n-1] = (x[n])^2$.
 - Find the impulse response. Is it useful?
 - Find $y[n]$ for all n when $x[n] = \cos(\omega n)$.
- An LTI system is governed by the difference equation $y[n] = \alpha y[n-1] + x[n]$. Let $x[n] = 2u[n]$. Find all $\alpha \in (0, 1)$ so that $y[1] \geq 1$. The system is in a relaxed state ($y[n] = 0$ for $n < 0$).
- Consider the difference equation $y[n+2] + \frac{1}{2}y[n+1] - \frac{1}{2}y[n] = x[n+1] + x[n]$.
 - Find the impulse response and the transfer function.
 - Is the system described by this equation causal? Is it BIBO stable?
 - Find $y[n]$ for all $n \geq 0$ when $x[n] = 4^{-n}u[n]$, $y[-1] = y[-2] = 1$.
- Write down the transfer function of the system implemented in the diagram below.



- (Bonus) Find the equivalent resistance of the infinite resistor network shown. All resistors have the same value (call it R). *Hint: Recast it as difference equation.*

